Ph. D. Thesis Defence

Structure-Preserving Finite Volume Schemes for Weak and Measure-Valued Solutions of Conservation Laws

By

Mr. Amogh S Krishnamurthy

Abstract

Hyperbolic conservation laws play a key role in modelling several fundamental phenomena in the real world, typically encoding the key principles of the conservation of mass, momentum and energy in the form of partial differential equations. Recent breakthroughs using convex integration techniques have shown that, contrary to expectations, entropy weak solutions to conservation laws need not be unique. This led to the proposal of a framework for dissipative measure-valued (DMV) solutions as an alternative to entropy weak solutions. The class of DMV solutions can be seen as the largest possible class of solutions to a given conservation law in which the classical solutions remain stable. Further, DMV solutions naturally arise as limits of certain suitable numerical schemes. Since conservation laws model the real physical world, sufficient care must be taken when designing numerical schemes to ensure that the key underlying physical properties are preserved at the discrete level. Further, the schemes should be robust enough to capture the solutions across all asymptotic regimes. In this talk, we present various structure-preserving finite-volume schemes that reproduce the key physical properties of the continuous system at the discrete level. These properties include, inter alia, conservation, positivity, energy stability, well-balancing and consistency à la Lax's equivalence theorem. For the Euler system with the congestion pressure law, a natural condition is imposed that the density is strictly less than 1, as the pressure law under consideration has a singularity at 1. We demonstrate that our scheme preserves this bound at the discrete level. Furthermore, through extensive numerical experiments, we show that the proposed scheme is robust across all regimes. For the rotating shallow water (RSW) equations, we demonstrate that our proposed finite-volume scheme preserves geostrophic steady states (jets in the rotational frame), rendering it well-balanced. In addition, we also prove the weak convergence of the numerical solutions to a DMV solution of the RSW system. For the compressible barotropic Euler system, we present two distinct finite-volume schemes. We show the convergence of both schemes towards a DMV solution of the Euler system. Furthermore, one of the schemes we present is asymptotic preserving and generates a classical solution of the incompressible Euler system in the zero-Mach-number limit. Using the relative energy as a tool, rigorous error estimates and convergence rates are derived, with the strong solution of the target system serving as a reference in both the compressible and incompressible regimes. Extensive numerical studies validate the theoretical expectations and the performance of the methods.

Date: March 16, 2026
Time: 4.00 PM, Venue: MSB Harish Chandra